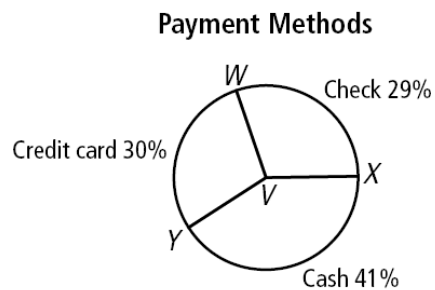


### Attendance Problems

1. What percent of 60 is 18?

2. What number is 44% of 6?

3. Find  $m\angle WVX$ .



- I can apply properties of arcs.
- I can apply properties of chords.

Vocabulary			
Arc	Adjacent Arcs	Semicircle	Minor arc
congruent arcs	Central angle	Major Arc	

**Common Core**

**CC.9-12.G.C.2** Identify and describe relationships among inscribed angles, radii, and chords.

**CC.12.2: CC.9-12.G.CO.12** Make formal geometric constructions with a variety of tools and methods.

A **central angle** is an angle whose vertex is the center of a circle. An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

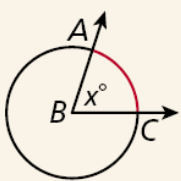
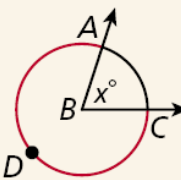
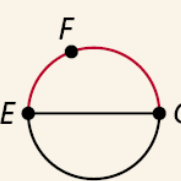
**Question:** What is the hidden math term?

**Answer:** Circle graph.



"Transform adversity into an enjoyable challenge!"—*Author and Researcher, Mihalyi Csikszent*

**Arcs and Their Measure**

ARC	MEASURE	DIAGRAM
A <b>minor arc</b> is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\widehat{AC} = m\angle ABC = x^\circ$	
A <b>major arc</b> is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to $360^\circ$ minus the measure of its central angle. $m\widehat{ADC} = 360^\circ - m\angle ABC$ $= 360^\circ - x^\circ$	
If the endpoints of an arc lie on a diameter, the arc is a <b>semicircle</b> .	The measure of a semicircle is equal to $180^\circ$ . $m\widehat{EFG} = 180^\circ$	

**Writing Math**

Minor arcs may be named by two points. Major arcs and semicircles must be named by three points.

**1 Data Application**

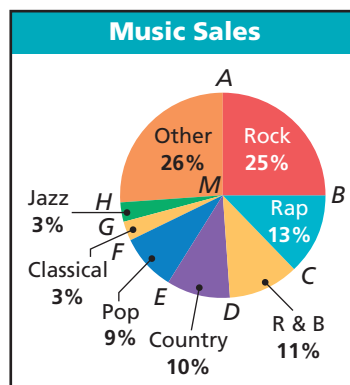
The circle graph shows the types of music sold during one week at a music store.

Find  $m\widehat{BC}$ .

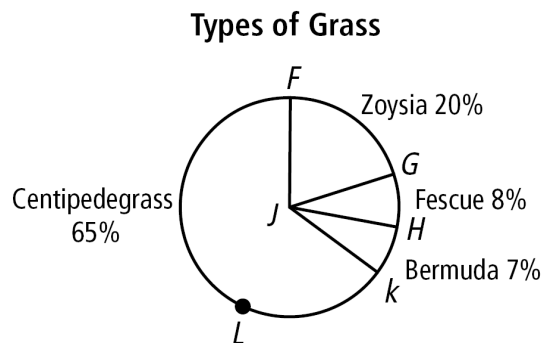
$$m\widehat{BC} = m\angle BMC$$

$$m\angle BMC = 0.13(360^\circ) = 46.8^\circ$$

*$m$  of arc =  $m$  of central  $\angle$ .  
Central  $\angle$  is 13% of the  $\odot$ .*



**Example 1.** The circle graph shows the types of grass planted in the yards of one neighborhood. Find  $m\widehat{KLF}$ .

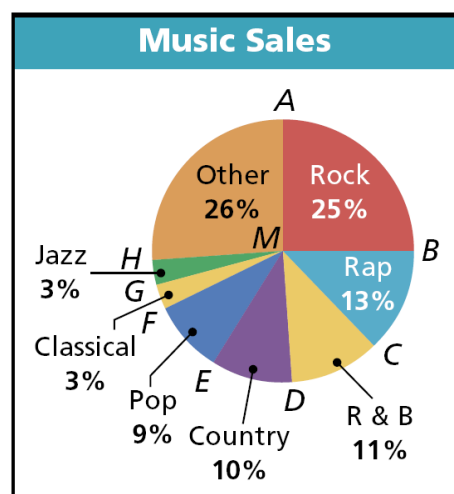


**Guided Practice.** Use the graph to find each of the following.

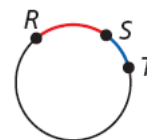
4.  $m\angle FMC$

5.  $m\widehat{AHB}$

6.  $m\angle EMD$



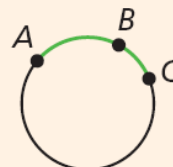
**Adjacent arcs** are arcs of the same circle that intersect at exactly one point.  
 $\widehat{RS}$  and  $\widehat{ST}$  are adjacent arcs.



### Postulate 11-2-1 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$$



## 2 Using the Arc Addition Postulate

Find  $m\widehat{CDE}$

$$m\widehat{CD} = 90^\circ$$

$$m\angle DFE = 18^\circ$$

$$m\widehat{DE} = 18^\circ$$

$$m\widehat{CE} = m\widehat{CD} + m\widehat{DE}$$

$$= 90^\circ + 18^\circ = 108^\circ$$

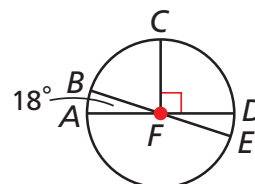
$$m\angle CFD = 90^\circ$$

Vert.  $\angle$  Thm.

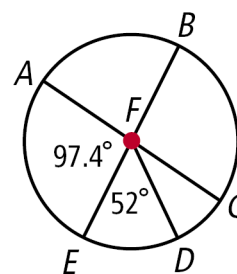
$$m\angle DFE = 18^\circ$$

Arc Add. Post.

Substitute and simplify.



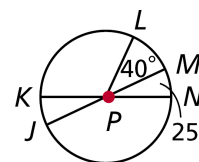
**Example 2.** Find  $m\widehat{BD}$ .



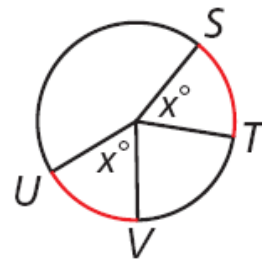
**Guided Practice.** Find each measure.

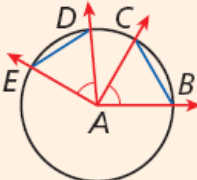
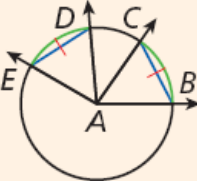
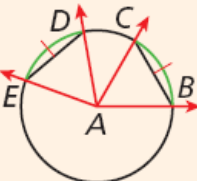
7.  $m\widehat{JKL}$

8.  $m\widehat{LJN}$



Within a circle or congruent circles, **congruent arcs** are two arcs that have the same measure. In the figure  $\widehat{ST} \cong \widehat{UV}$ .

**Theorem 11-2-2**

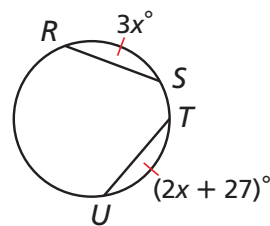
THEOREM	HYPOTHESIS	CONCLUSION
In a circle or congruent circles:		
(1) Congruent central angles have congruent chords.	 $\angle EAD \cong \angle BAC$	$\overline{DE} \cong \overline{BC}$
(2) Congruent chords have congruent arcs.	 $\overline{ED} \cong \overline{BC}$	$\widehat{DE} \cong \widehat{BC}$
(3) Congruent arcs have congruent central angles.	 $\widehat{ED} \cong \widehat{BC}$	$\angle DAE \cong \angle BAC$

**3 Applying Congruent Angles, Arcs, and Chords**

Find each measure.

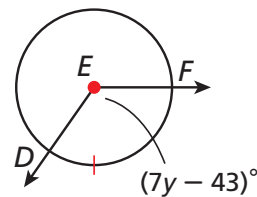
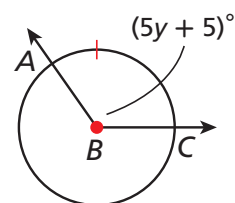
**A**  $\overline{RS} \cong \overline{TU}$ . Find  $m\widehat{RS}$ .

$$\begin{aligned} \widehat{RS} &\cong \widehat{TU} && \cong \text{ chords have } \cong \text{ arcs.} \\ m\widehat{RS} &= m\widehat{TU} && \text{Def. of } \cong \text{ arcs} \\ 3x &= 2x + 27 && \text{Substitute the given measures.} \\ x &= 27 && \text{Subtract } 2x \text{ from both sides.} \\ m\widehat{RS} &= 3(27) && \text{Substitute 27 for } x. \\ &= 81^\circ && \text{Simplify.} \end{aligned}$$

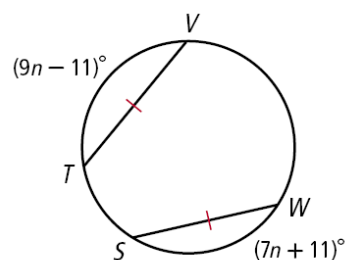


**B**  $\odot B \cong \odot E$ , and  $\widehat{AC} \cong \widehat{DF}$ . Find  $m\angle DEF$ .

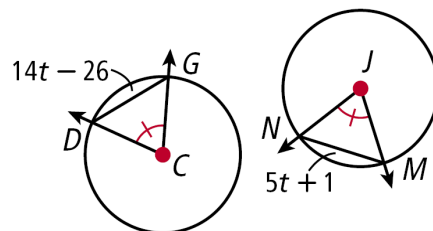
$$\begin{aligned} \angle ABC &\cong \angle DEF && \cong \text{ arcs have } \cong \text{ central } \angle\text{s.} \\ m\angle ABC &= m\angle DEF && \text{Def. of } \cong \angle\text{s} \\ 5y + 5 &= 7y - 43 && \text{Substitute the given measures.} \\ 5 &= 2y - 43 && \text{Subtract } 5y \text{ from both sides.} \\ 48 &= 2y && \text{Add 43 to both sides.} \\ 24 &= y && \text{Divide both sides by 2.} \\ m\angle DEF &= 7(24) - 43 && \text{Substitute 24 for } y. \\ &= 125^\circ && \text{Simplify.} \end{aligned}$$

**Example 3. Find each measure.**

**A.**  $\overline{TV} \cong \overline{WS}$ . Find  $m\widehat{WS}$ .

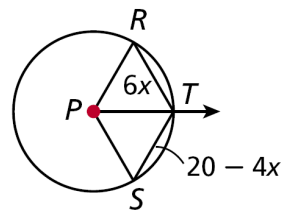


B.  $\odot C \cong \odot J$ , and  $m\angle GCD \cong m\angle NJM$ . Find NM.



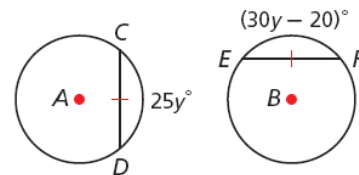
**Guided Practice.** Find each measure.

9.  $\overline{PT}$  bisects  $\angle RPS$ . Find RT.





10.  $\odot A \cong \odot B$  &  $\overline{CD} \cong \overline{EF}$ . Find  $m\widehat{CD}$ .



12-2 Arcs and chords: (p 807) 21, 23, 25, 27, 29, 30.

### Theorems

THEOREM	HYPOTHESIS	CONCLUSION
<b>11-2-3</b> In a circle, if a radius (or diameter) is perpendicular to a chord, then it bisects the chord and its arc.	 $\overline{CD} \perp \overline{EF}$	$\overline{CD}$ bisects $\overline{EF}$ and $\widehat{EF}$ .
<b>11-2-4</b> In a circle, the perpendicular bisector of a chord is a radius (or diameter).	 $\overline{JK}$ is $\perp$ bisector of $\overline{GH}$ .	$\overline{JK}$ is a diameter of $\odot A$ .

**4****Using Radii and Chords**Find  $BD$ .**Step 1** Draw radius  $\overline{AD}$ .

$$AD = 5$$

*Radii of a  $\odot$  are  $\cong$ .***Step 2** Use the Pythagorean Theorem.

$$CD^2 + AC^2 = AD^2$$

$$CD^2 + 3^2 = 5^2$$

*Substitute 3 for AC and 5 for AD.*

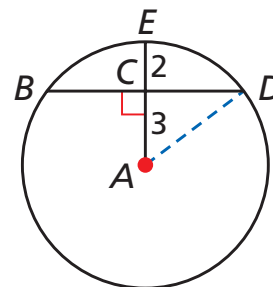
$$CD^2 = 16$$

*Subtract  $3^2$  from both sides.*

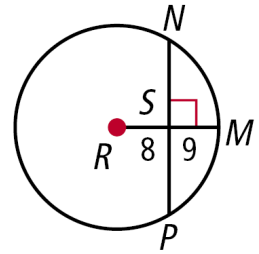
$$CD = 4$$

*Take the square root of both sides.***Step 3** Find  $BD$ .

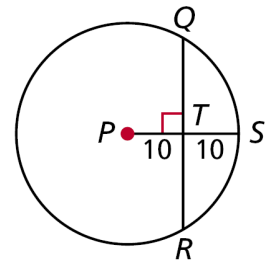
$$BD = 2(4) = 8$$

 *$\overline{AE} \perp \overline{BD}$ , so  $\overline{AE}$  bisects  $\overline{BD}$ .*

**Example 4.** Find NP.



**11. Guided Practice.** Find QR. Round your final answer to the nearest tenth.



**12-2 Arcs and chords:** (p 807) 21, 23, 25, 27, 29-35, 37, 38, 40, 45-47.

